MOTOR-DA OO Design

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Note that the current operation system applies Laplacian operators to instead of , as it calls ctl%state before applying the Laplacian operators. Thus, it is a little different from the following formulation. The current formulation is

It requires additional transformation .

For the variational analysis, we are solving the following minimization problem,

Where is the control variable, is the Laplacian operator, is the background error covariance, is the observation error covariance. Usually, we introduce a change of variable to avoid the . Let us assume and is symmetric. Define a new variable,

The minimization problem becomes,

When , it reduces to Laplacian only analysis when . Thus, this is a generalized form of MOTOR-DA variational analysis for either 3DVAR or 4DVAR analysis.

Now let us calculate the derivative of this cost function.

When we added mask fields in state variables, the background terms need special attention. Consistent with a 4DVar implementation, we will count all variables in the cost function.

**4DVar Implementation, see the pptx document: MOTOR-DA 4DVar Design under 深圳中心/2024/项目课题/基金委/**

The above formula of is a single time frame one.

**An adjoint derivation of a 4th order Runge-Kutta scheme:**

Consider the 4th order Runge-Kutta (RK4) scheme:

with

For a given 4DVar problem using a RK4,

Its Lagrangian function is

Applying a differential operator,

where

Similarly,

Thus,

The adjoint equation is

and for , we force

And we have

From the last term, we can see that it requires calls of the adjoints of 10 times comparing to the RK4 4 times of the forward operator. It is too expensive to run.

**An adjoint derivation of a multistep time integration scheme:**

Consider a multistep time integration scheme,

The corresponding Lagrangian function as a functional of is

Here assuming the background covariance applied to the only.

Look at the last term of the above equation, take an example of ,

with assuming . Thus the last term can be written as,

with assuming . Therefore,

It requires initial conditions of

and have

For solving

Or

Then we have

So

and for any ,

MOTOR-DA dynamic downscaling: